

SOLUCIONES FRACCIONES ALGEBRAICAS

1º) Opera y simplifica:

$$a) \frac{x-x^2}{1-x^2} + \frac{1+x}{x^2+2x+1} - \frac{1-2x}{1-x} = \frac{x(1-x)}{(1+x)(1-x)} + \frac{1+x}{(1+x)^2} - \frac{1-2x}{1-x} = \frac{x}{1+x} + \frac{1}{1+x} - \frac{1-2x}{1-x} = \frac{x(1-x)+1-x-(1-2x)(1+x)}{(1+x)(1-x)} =$$

$$= \frac{x-x^2+1-x-(1+x-2x-2x^2)}{(1+x)(1-x)} = \frac{x^2+x}{(1+x)(1-x)} = \frac{x(1+x)}{(1+x)(1-x)} = \frac{x}{1-x}$$

$$b) \frac{x}{x-2} - \frac{x}{x-1} - \frac{x}{x^2-3x+2} = \frac{x}{x-2} - \frac{x}{x-1} - \frac{x}{(x-2)(x-1)} = \frac{x(x-1)-x(x-2)-x}{(x-2)(x-1)} = \frac{x^2-x-x^2+2x-x}{(x-2)(x-1)} = \frac{0}{(x-2)(x-1)} = 0$$

$$c) \left(\frac{x^2+x-6}{x-1} \right) \left(\frac{x^2-2x+1}{(x+3)^2} \right) : \left(\frac{x-2}{x+3} \right) = \left(\frac{(x-2)(x+3)}{x-1} \right) \left(\frac{(x-1)^2}{(x+3)^2} \right) : \left(\frac{x-2}{x+3} \right) = \frac{(x-2)(x+3)(x-1)^2(x+3)}{(x-1)(x+3)^2(x-2)} = x-1$$

$$d) \frac{x-y}{x+y} - \frac{3x^2-y^2}{x^2+2xy+y^2} = \frac{(x-y)(x+y)-3x^2+y^2}{(x+y)^2} = \frac{x^2-y^2-3x^2+y^2}{(x+y)^2} = -\frac{2x^2}{(x+y)^2}$$

$$e) \left(\frac{5c}{5c+3d} + \frac{3d}{5c-3d} \right) : \left(\frac{5c}{5c-3d} - \frac{3d}{5c+3d} \right) = \frac{5c(5c-3d)+3d(5c+3d)}{25c^2-9d^2} : \frac{5c(5c+3d)-3d(5c-3d)}{25c^2-9d^2} = \frac{5c(5c-3d)+3d(5c+3d)}{5c(5c+3d)-3d(5c-3d)}$$

$$= \frac{25c^2-15cd+15cd+9d^2}{25c^2+15cd-15cd+9d^2} = 1$$

$$f) \frac{x^2-y^2}{x^2+xy} \cdot \frac{3x}{x-y} = \frac{(x+y)(x-y) \cdot 3x}{x(x+y)(x-y)} = 3$$

$$g) \frac{x^2-1}{x^2-2x} - \frac{3-2x^2}{x^2+2x} + \frac{3x^2+1}{x^2-4} - \frac{3x+4}{x} = \frac{x^2-1}{x(x-2)} - \frac{3-2x^2}{x(x+2)} + \frac{3x^2+1}{(x+2)(x-2)} - \frac{3x+4}{x} = \frac{(x^2-1)(x+2) - (3-2x^2)(x-2)}{x(x+2)(x-2)} +$$

$$+ \frac{(3x^2+1) \cdot x - (3x+4)(x^2-4)}{x(x+2)(x-2)} = \frac{x^3+2x^2-x-2-3x+6+2x^3-4x^2+3x^3+x-3x^3+12x-4x^2+16}{x(x+2)(x-2)} = \frac{3x^3-6x^2+9x+20}{x(x+2)(x-2)}$$

$$h) \frac{x-x^2}{1-x^2} + \frac{1+x}{x^2+2x+1} - \frac{1-2x}{1-x} = \frac{x(1-x)}{(1+x)(1-x)} + \frac{1+x}{(1+x)^2} - \frac{1-2x}{1-x} = \frac{x}{1+x} + \frac{1}{1+x} - \frac{1-2x}{1-x} = \frac{x(1-x)+1-x-(1-2x)(1+x)}{(1+x)(1-x)} =$$

$$= \frac{x-x^2+1-x-(1+x-2x-2x^2)}{(1+x)(1-x)} = \frac{x^2+x}{(1+x)(1-x)} = \frac{x(1+x)}{(1+x)(1-x)} = \frac{x}{1-x}$$

$$i) \left(x + \frac{x}{3} + \frac{2}{3}x \right) \cdot \frac{3x}{2(x+1)(x-1)} = \frac{3x+x+2x}{3} \cdot \frac{3x}{2(x+1)(x-1)} = 2x \cdot \frac{3x}{2(x+1)(x-1)} = \frac{3x^2}{(x+1)(x-1)}$$

$$j) \left(\frac{a^2-a}{a-5} \right) \left(\frac{a^2-a-20}{a^2-25} \right) : \left(\frac{a^2+2a-8}{a^2-3a+2} \right) = \frac{a(a-1)}{a-5} \cdot \frac{(a+4)(a-5)}{(a+5)(a-5)} : \frac{(a+4)(a-2)}{(a-1)(a-2)} = \frac{a(a-1)^2(a+4)(a-2)}{(a+5)(a-5)(a+4)(a-2)} = \frac{a(a-1)^2}{a^2-25}$$

$$k) \left(\frac{a+b}{a-b} - \frac{a-b}{a+b} \right) : \left(\frac{a-b}{a+b} + \frac{a+b}{a-b} \right) = \frac{(a+b)^2 - (a-b)^2}{a^2 - b^2} : \frac{(a-b)^2 + (a+b)^2}{a^2 - b^2} = \frac{(a+b)^2 - (a-b)^2}{(a-b)^2 + (a+b)^2} = \frac{a^2 + 2ab + b^2 - a^2 + 2ab - b^2}{a^2 - 2ab + b^2 + a^2 + 2ab + b^2}$$

$$= \frac{4ab}{2a^2 + 2b^2} = \frac{2ab}{a^2 + b^2}$$

$$l) \frac{x^4 + y^4}{(x^2 - y^2)^2} + \frac{4xy}{x^2 - y^2} - \frac{(x-y)^2}{x^2 + 2xy + y^2} = \frac{x^4 + y^4}{(x^2 - y^2)^2} + \frac{4xy(x^2 - y^2)}{(x^2 - y^2)^2} - \frac{(x-y)^2 \cdot (x-y)^2}{(x^2 - y^2)^2} = \frac{x^4 + y^4}{(x^2 - y^2)^2} + \frac{4x^3y - 4xy^3}{(x^2 - y^2)^2}$$

$$- \frac{x^4 - 2x^3y + x^2y^2 - 2x^3y + 4x^2y^2 - 2xy^3 + y^2x^2 - 2xy^3 + y^4}{(x^2 - y^2)^2} = \frac{8x^3y - 6x^2y^2}{(x^2 - y^2)^2} = \frac{2x^2y(4x - 3y)}{(x^2 - y^2)^2}$$

$$m) \frac{a}{2b} - \frac{a-1}{3b} + \frac{a-b}{6b} - \frac{b}{4} = \frac{6a - 4a + 4 + 2a - 2b - 3b^2}{12b} = \frac{4a - 2b - 3b^2 + 4}{12b}$$

$$n) \frac{3-x}{x} + \frac{2x}{x-1} - \frac{x-1}{3x} = \frac{3(3-x)(x-1) + 2x \cdot 3x - (x-1)^2}{3x(x-1)} = \frac{9x - 9 - 3x^2 + 3x + 6x^2 - x^2 + 2x - 1}{3x(x-1)} = \frac{2x^2 + 14x - 10}{3x(x-1)}$$

$$ñ) \frac{x-2}{x-1} + \frac{x(x-1)}{x-2} - \frac{x-1}{2} = \frac{2(x-2)^2 + 2x(x-1)^2(x-1)^2(x-2)}{2(x-1)(x-2)} = \frac{2(x^2 - 4x + 4) + 2x(x^2 - 2x + 1) - (x^2 - 2x + 1)(x-2)}{2(x-1)(x-2)}$$

$$= \frac{2x^2 - 8x + 8 + 2x^3 - 4x^2 + 2x - x^3 + 2x^2 + 2x^2 - 4x - x + 2}{2(x-1)(x-2)} = \frac{x^3 + 2x^2 - 11x + 10}{2(x-1)(x-2)}$$

$$o) \frac{a+b}{a-b} + \frac{2-a}{b-a} - \frac{b^2}{a^2 - b^2} = \frac{a+b}{a-b} + \frac{a-2}{a-b} - \frac{b^2}{a^2 - b^2} = \frac{(a+b)^2 + (a-2)(a+b) - b^2}{a^2 - b^2} = \frac{a^2 + 2ab + b^2 + a^2 + ab - 2a - 2b - b^2}{a^2 - b^2} =$$

$$= \frac{2a^2 + 3ab - 2a - 2b}{a^2 - b^2}$$

$$p) \frac{x-2}{6x+6} - \frac{x+2}{2x+2} + \frac{3-x}{4x+4} = \frac{x-2}{6(x+1)} - \frac{x+2}{2(x+1)} + \frac{3-x}{4(x+1)} = \frac{2x-4-6x-12+9-3x}{12(x+1)} = \frac{-7x-7}{12(x+1)} = \frac{-7(x+1)}{12(x+1)} = -\frac{7}{12}$$

$$q) \frac{x-y}{x+y} - \frac{3x^2 - y^2}{x^2 + 2xy + y^2} = \frac{(x-y)(x+y) - 3x^2 + y^2}{(x+y)^2} = \frac{x^2 - y^2 - 3x^2 + y^2}{(x+y)^2} = -\frac{2x^2}{(x+y)^2}$$

$$r) \frac{x}{x^2 - y^2} + \frac{1}{(x-y)^2} - \frac{1}{(x+y)^2} - \frac{4xy}{(x^2 - y^2)^2} = \frac{x(x^2 - y^2)}{(x^2 - y^2)^2} + \frac{(x+y)^2}{(x^2 - y^2)^2} - \frac{(x-y)^2}{(x^2 - y^2)^2} - \frac{4xy}{(x^2 - y^2)^2} = \frac{x(x^2 - y^2)}{(x^2 - y^2)^2} + \frac{x^2 + 2xy + y^2}{(x^2 - y^2)^2} -$$

$$- \frac{x^2 - 2xy + y^2}{(x^2 - y^2)^2} - \frac{4xy}{(x^2 - y^2)^2} = \frac{x(x^2 - y^2) + x^2 + 2xy + y^2 - x^2 + 2xy - y^2 - 4xy}{(x^2 - y^2)^2} = \frac{x(x^2 - y^2)}{(x^2 - y^2)^2} = \frac{x}{x^2 - y^2}$$

$$s) (x+1) \left(\frac{(x+1)^2}{x^2 - 1} + x - 1 \right) = (x+1) \left(\frac{x+1}{x-1} + x - 1 \right) = (x+1) \left(\frac{x+1 + x(x-1) - (x-1)}{x-1} \right) = \frac{(x+1)(x^2 - x + 2)}{x-1}$$

$$t) \frac{(x+y)^2 - z^2}{x^2 - (y+z)^2} \cdot \frac{x-y-z}{x+y-z} = \frac{(x+y+z)(x+y-z)}{(x+y+z)(x-y-z)} \cdot \frac{x-y-z}{x+y-z} = 1$$

$$u) \left(\frac{x-y}{y} \right) \left(\frac{x+y}{x} \right) \left(\frac{xy}{x^2+y^2} \right) = \frac{x^2-y^2}{xy} \cdot \frac{x^2+y^2}{xy} \cdot \frac{xy}{x^2+y^2} = \frac{x^2-y^2}{xy}$$

$$v) \frac{(a-1)^2}{x^2-1} \cdot \frac{a^2-1}{(x-1)^2} = \frac{(a-1)(a-1)(x-1)(x-1)}{(x+1)(x-1)(a+1)(a-1)} = \frac{(a-1)(x-1)}{(x+1)(a+1)}$$

$$w) \left[\left(\frac{x}{y} - y \right) \left(\frac{x}{y} + y \right) \right] : \left(\frac{x}{x^2-y^4} - \frac{1}{x-y^2} \right) = \left[\left(\frac{x-y^2}{y} \right) \left(\frac{x+y^2}{y} \right) \right] : \left(\frac{x-(x+y^2)}{x^2-y^4} \right) = -\frac{(x^2-y^4)^2}{y^4}$$

$$x) \left(\frac{2x-8x^2}{16x^2-1} \cdot \frac{16x^2+1+8x}{6x^2} \right) : \frac{4x-16x^2+2}{24x^2-12x} = \left(\frac{-2x(4x-1)}{(4x+1)(4x-1)} \cdot \frac{(4x+1)^2}{6x^2} \right) : \frac{-2(2x-1)(4x+1)}{12x(2x-1)} = \frac{-(4x+1)}{3x} \cdot \frac{-6x}{(4x+1)} = 2$$

$$y) \left(\frac{x^2-6x+9}{x^2-x} \cdot \frac{x^2-4x+3}{x^2-4x+4} \right) : \frac{x^2-5x+6}{x^2-2x+1} = \frac{(x-3)^2(x-2)^2(x-1)^2}{x(x-1)(x-3)(x-1)(x-3)(x-2)} = \frac{x-2}{x}$$

2º) Simplifica:

$$a) \frac{x^3+x^2-6x}{x^3+4x^2+x-6} = \frac{x(x-2)(x+3)}{(x-1)(x+2)(x+3)} = \frac{x(x-2)}{(x-1)(x+2)}$$

$$b) \left(\frac{3a+3}{12-12a} \right) : \left(\frac{(a+1)^2}{a^2-1} \right) = \frac{3(a+1)}{12(1-a)} \cdot \frac{(a+1)(a-1)}{(a+1)^2} = -\frac{1}{4}$$

$$c) \frac{xy-2x-3y+6}{ay-by+2b-2a} = \frac{x(y-2)-3(y-2)}{a(y-2)-b(y-2)} = \frac{(x-3)(y-2)}{(a-b)(y-2)} = \frac{(x-3)}{(a-b)}$$